Full Action for an Electromagnetic Field with Electrical and Magnetic Charges

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Abstract

The paper offers the full action for an electromagnetic field with electrical and magnetic charges; it is marked, that it is hard to give an accurate formulation of Feynman laws for the calculation of the interaction cross-sections for electrically and magnetically charged particles on the base of offerd action within relativistic quantum field theory, simple partial case of a constant electromagnetic field is considered.

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I. INTRODUCTION

The papers [1], [2] by Dirac, in which the existence of magnetic charges was related to quantization of electrical charges, were followed by intensively searching for magnetic monopoles, however, the search has not yet led to success (see, for example, [3]). All the attempts of such search were complicated by the lack of a satisfactory theoretical description of interactions between ordinary electrical and magnetic charges. Moreover, Rohrlich in [4] stated the "falsehood of variational principle for the full theory of ... electrical and magnetic point charges". Later, Zwanziger in [5] proposed the full action including an arbitrary constant 4-vector n, with the electromagnetic field propagator being dependent of the arbitrary vector n; as a result, for a partial case of interaction between an electrical charge and a magnetic charge, when the dependence is not reduced, we come to an absurd result - the cross-section of interaction of the electrical and magnetic charges depends on the *arbitrary* constant vector. If everything were valid in Zwanziger's considerations, impossibility of the existence of magnetic charges would follow (or implication that something is wrong in relativistic quantum field theory). Hence, the full action construction for a system of interacting electrical and magnetic charges (and clarification of the cause of "failure" in efforts by Zwanziger) is the matter of principle.

The paper proposes the full action for an electromagnetic field with electrical and magnetic charges and mentions, in passing, that the expression for the Lagrangian of interaction between electrical and magnetic charges and an external field, used in the paper by Zwanziger [5] without discussions, is not correct.

We do not consider particles with both electrical and magnetic charges (compare with the statement by Weinberg in [6] that "a magnetic monopole cannot bear a normal charge").

The expressions below are written in Gaussian units. The denotations we use are close to that used in [7] and [8], in particular, the coordinate indexes of vectors and tensors have values from 0 to 3; a metrics is considered to be defined with diagonal metric tensor g: $g^{ik} = 0$ at $i \neq k$, $g^{00} = 1$, $g^{11} = g^{22} = g^{33} = -1$; the same upper and lower coordinate indexes of four-dimensional tensors always imply summation; if some indexes are not coordinate tensor indexes, the summation symbol is explicitly shown, if necessary; the four-dimensional coordinate is $x^i = (ct, \boldsymbol{r})$, $x_i = (ct, -\boldsymbol{r})$, $x^i x_i = c^2t^2 - \boldsymbol{r}^2$, where c is the light speed constant, t is time, $\boldsymbol{r}$ is three-dimensional radius-vector.

II. MAGNETIC MONOPOLE IN CLASSIC FIELD THEORY

It would appear reasonable that the properties of magnetic charges and the electromagnetic field generated by them would be similar to the properties of electrical charges and the electromagnetic field generated by them: in particular, the force lines of the magnetic field generated by magnetic charges start from/end in the magnetic charges, while the force lines of the electrical field generated by currents of magnetic charges are closed, i.e. the existence of magnetic charges, in a sense, restores *symmetry* between the magnetic and electrical fields. Apparently, the equations describing the electromagnetic field generated by magnetic charges (currents) should be similar to the equations describing the electromagnetic field generated by electrical charges (currents). At the same time, in view of the topological difference between the electromagnetic field generated by magnetic charges and the one generated by electrical charges they should be described *separately* in equations.

The electromagnetic field generated by the current density of electrical charges $\{e_a\}$ – j_e,– can be described using the antisymmetric four-dimensional tensor of the second rank, F_{A_e}, which can be represented via the four-dimensional vector potential, $A_e^i \equiv (\varphi_e, \boldsymbol{A}_e)$ in the following form:

$$F_{A_e}^{ik} \equiv \frac{\partial A_e^k}{\partial x_i} - \frac{\partial A_e^i}{\partial x_k} = \partial^i A_e^k - \partial^k A_e^i. \tag{1}$$

The relation between components of tensor F_{A_e} and components of (three-dimensional) vectors of electrical field $\boldsymbol{E}$ and magnetic field $\boldsymbol{H}$ is described as

$$E_e^\alpha = -F_{A_e}^{0\alpha} \quad (\alpha = 1,2,3)\,; \quad H_e^1 = -F_{A_e}^{23}, \quad H_e^2 = -F_{A_e}^{31}, \quad H_e^3 = -F_{A_e}^{12} \tag{2}$$

or

$$F_{A_e}^{ik} = \begin{pmatrix} 0 & -E_e^1 & -E_e^2 & -E_e^3 \\ E_e^1 & 0 & -H_e^3 & H_e^2 \\ E_e^2 & H_e^3 & 0 & -H_e^1 \\ E_e^3 & -H_e^2 & H_e^1 & 0 \end{pmatrix} \qquad F_{A_e\,ik} = \begin{pmatrix} 0 & E_e^1 & E_e^2 & E_e^3 \\ -E_e^1 & 0 & -H_e^3 & H_e^2 \\ -E_e^2 & H_e^3 & 0 & -H_e^1 \\ -E_e^3 & -H_e^2 & H_e^1 & 0 \end{pmatrix}. \tag{3}$$

An ordinary Maxwell's equation system describing an electromagnetic field (in vacuum) generated by the current density of electrical charges, j_e can be written as

$$\frac{\partial F_{A_e}^{ik}}{\partial x_l} + \frac{\partial F_{A_e}^{kl}}{\partial x_i} + \frac{\partial F_{A_e}^{li}}{\partial x_k} = 0, \tag{4a}$$

$$\frac{\partial F^{ik}_{A_e}}{\partial x_k} = -\frac{4\pi}{c} j^i_e. \tag{4b}$$

Since the result of action of operator $\frac{\partial^2}{\partial x_i \partial x_k}$ (symmetric with respect to indexes i, k) on antisymmetric tensor $F^{ik}_{A_e}$ (similar to any convolution of symmetric tensor with the antisymmetric one) identically equals 0, the continuity equation describing the charge conservation law follows from (4b):

$$\frac{\partial j^i_e}{\partial x^i} = 0. \tag{5}$$

In quantum field theory, equation (4b) is replaced, according to definition (1), by the following equation:

$$-\partial_k \partial^k A^i_e + \partial_k \partial^i A^k_e \equiv \Box A^i_e + \partial_k \partial^i A^k_e = -\frac{4\pi}{c} j^i_e. \tag{6}$$

In the equation (4a) instead of tensor F_{A_e} *pseudo-tensor*

$$F^{\star\, ik}_{A_e} \equiv \frac{1}{2!}\, e^{iklm} F_{A_e\, lm}, \tag{7}$$

dual to tensor F_{A_e}, may be used, where e^{iklm} is the absolutely antisymmetric, unit, four-dimensional pseudo-tensor of the fourth rank with weight $W = +1$, which components change their signs with permutation of any two indexes and

$$e^{0123} \equiv +1, \tag{8}$$

by definition (pseudo-tensor e_{iklm} has weight $W = -1$ and $e_{0123} = -1$). With x coordinate system replaced by $\bar{x}$ coordinate system (caused by changes in the basis of n-dimensional vector space), components of pseudo-tensor T of weight W, which is r-times contra-variant and s-times co-variant, are transformed according to the law (see, for example, [9]):

$$\bar{T}^{k_1 k_2 \ldots k_r}_{k'_1 k'_2 \ldots k'_s} \equiv \frac{\partial \bar{x}^{k_1}}{\partial x^{i_1}} \frac{\partial \bar{x}^{k_2}}{\partial x^{i_2}} \cdots \frac{\partial \bar{x}^{k_r}}{\partial x^{i_r}} \frac{\partial x^{i'_1}}{\partial \bar{x}^{k'_1}} \frac{\partial x^{i'_2}}{\partial \bar{x}^{k'_2}} \cdots \frac{\partial x^{i'_s}}{\partial \bar{x}^{k'_s}} T^{i_1 i_2 \ldots i_r}_{i'_1 i'_2 \ldots i'_s} J^W. \tag{9}$$

In contrast to the ordinary tensor's component transformation, there is a factor in the form of Jacobian of coordinate transformation

$$J = \frac{\partial\left(x^0, x^1, \ldots, x^{n-1}\right)}{\partial\left(\bar{x}^0, \bar{x}^1, \ldots, \bar{x}^{n-1}\right)} \qquad |J| = \sqrt{-g} = 1 \tag{10}$$

of power W in equation (9). By virtue of such factor components of tensor F_{A_e} and (dual to them) components of pseudo-tensor $F^\star_{A_e}$, dual to F_{A_e}, are transformed in the same way

with coordinate system replacement. The weight of the product of two tensors equals the summarized weight of tensors of each factor and convolution of each pair of indexes, consisting of the identical upper and lower indexes, does not change the weight of tensor; all pseudo-tensors considered in the paper have weight ± 1; in particular, a magnetic charge is a pseudo-scalar, see below, and, hence, we are not far from the fore-quoted Weinberg statement.

The left-hand part of equation (4a) represents by itself a tensor of the third rank, which is antisymmetric in all indexes. Having lowered indexes of this tensor, multiplied by e^{mlik} and performed convolution with respect to three pairs of identical indexes, we obtain equation

$$\frac{\partial F^{\star\, ml}_{A_e}}{\partial x_l} = 0. \tag{11}$$

The relation between components of pseudo-tensor $F^{\star}_{A_e}$, which is dual to F_{A_e}, and components of (three-dimensional) vectors of electrical field $\boldsymbol{E}$ and magnetic field $\boldsymbol{H}$ is described as

$$H^{\alpha}_{e} = -F^{\star\, 0\alpha}_{A_e} \quad (\alpha = 1, 2, 3)\,; \quad E^1_e = -F^{\star\, 23}_{A_e}, \quad E^2_e = -F^{\star\, 31}_{A_e}, \quad E^3_e = -F^{\star\, 12}_{A_e} \tag{12}$$

or

$$F^{\star\, ik}_{A_e} = \begin{pmatrix} 0 & -H^1_e & -H^2_e & -H^3_e \\ H^1_e & 0 & -E^3_e & E^2_e \\ H^2_e & E^3_e & 0 & -E^1_e \\ H^3_e & -E^2_e & E^1_e & 0 \end{pmatrix} \qquad F^{\star}_{A_e\, ik} = \begin{pmatrix} 0 & H^1_e & H^2_e & H^3_e \\ -H^1_e & 0 & -E^3_e & E^2_e \\ -H^2_e & E^3_e & 0 & -E^1_e \\ -H^3_e & -E^2_e & E^1_e & 0 \end{pmatrix}, \tag{13}$$

i.e. in pseudo-tensor $F^{\star}_{A_e}$, as compared to tensor F_{A_e}, components of (three-dimensional) vectors of the electrical and magnetic fields change places. So, it would appear reasonable, that the electromagnetic field generated by the current density of magnetic charges $\{\mu_b\}$ – j_μ could be described using the antisymmetric four-dimensional pseudo-tensor of the second rank, F_{B_μ}, which is structurally similar to pseudo-tensor $F^{\star}_{A_e}$ and described via four-dimensional pseudovector-potential $B^i_\mu \equiv (\psi_\mu, \boldsymbol{B}_\mu)$ in the following way:

$$F^{ik}_{B_\mu} \equiv \frac{\partial B^k_\mu}{\partial x_i} - \frac{\partial B^i_\mu}{\partial x_k} = \partial^i B^k_\mu - \partial^k B^i_\mu. \tag{14}$$

Then, instead of equations (4)-(6), (11), describing the electromagnetic field (in vacuum), generated by the current density of magnetic charges j_μ, one could use the following

equations:

$$\frac{\partial F^{ik}_{B_\mu}}{\partial x_l}+\frac{\partial F^{kl}_{B_\mu}}{\partial x_i}+\frac{\partial F^{li}_{B_\mu}}{\partial x_k}=0, \tag{15a}$$

$$\frac{\partial F^{ik}_{B_\mu}}{\partial x_k}=-\frac{4\pi}{c}j^i_\mu. \tag{15b}$$

$$\frac{\partial j^i_\mu}{\partial x^i}=0, \tag{16}$$

$$-\partial_k\partial^k B^i_\mu+\partial_k\partial^i B^k_\mu=\Box B^i_\mu+\partial_k\partial^i B^k_\mu=-\frac{4\pi}{c}j^i_\mu, \tag{17}$$

$$\frac{\partial F^{\star\, ml}_{B_\mu}}{\partial x_l}=0. \tag{18}$$

To obtain a closed equation system, describing the electromagnetic field together with electrical and magnetic charges, equations (4b) and (15b) must be complemented with motion equations for electrical charges

$$m_a\, c\,\frac{du^i_a}{ds}=\frac{e_a}{c}\left(F^{ik}_{A_e}+F^{\star\, ik}_{B_\mu}\right)u_{a\,k} \tag{19}$$

and magnetic charges

$$m_b\, c\,\frac{du^i_b}{ds}=\frac{\mu_b}{c}\left(F^{\star\, ik}_{A_e}+F^{ik}_{B_\mu}\right)u_{b\,k}. \tag{20}$$

In equation (19), m_a is the a-th electrical charge's mass, $ds=\sqrt{dx^i dx_i}$,

$$u_a\equiv\frac{dx^i_a}{ds} \tag{21}$$

is the without size four-dimensional velocity of the a-th electrical charge. The same notations are used in equation (20). In the right-hand parts of equations (19), (20) in parentheses the electrical and magnetic fields, generated by electrical and magnetic charges, are summarized, according to the superposition law.

Equations (19), (20), (4b), (15b) can be obtained, basing on the requirement of

$$\delta\mathscr{A}=0 \tag{22}$$

for variation of the full action,

$$\begin{aligned}\mathscr{A}=&-\sum_a\int m_a c ds-\frac{1}{c^2}\int\left(A^i_e+A^i_\mu\right)j_{e\,i}\,d^4x-\frac{1}{16\pi c}\int F_{A_e\,ik}\,F^{ik}_{A_e}d^4x\\ &-\sum_b\int m_b c ds-\frac{1}{c^2}\int\left(B^i_\mu+B^i_e\right)j_{\mu\,i}\,d^4x-\frac{1}{16\pi c}\int F_{B_\mu\,ik}\,F^{ik}_{B_\mu}d^4x,\end{aligned} \tag{23}$$

if we vary trajectories of electrical charges, trajectories of magnetic charges, vector-potential A_e and pseudovector-potential B_μ (compare, for example, with [7], §23 and §30). Integration in the first and fourth terms in the right-hand part of equation (23) is performed with respect to the world lines between two given events, in the rest terms integration is performed with respect to four-dimensional volume; variations of functions are assumed to be equal zero on the integration domain boundary. To vary trajectories of electrical charge e_a, we use the expression, describing the current density of this charge via the delta-function of three-dimensional argument:

$$j^i_{e_a} = e_a \delta\left(\boldsymbol{r} - \boldsymbol{r}_a\right) \frac{dx^i}{dt}, \tag{24}$$

a similar expression for the current density of a magnetic charge

$$j^i_{\mu_b} = e_b \delta\left(\boldsymbol{r} - \boldsymbol{r}_b\right) \frac{dx^i}{dt} \tag{25}$$

is used to vary the magnetic charge trajectories;

$$j^i_e = \sum_a j^i_{e_a}, \quad j^i_\mu = \sum_b j^i_{\mu_b}. \tag{26}$$

Because of (15b) density of current j_μ is a pseudo-vector and, in consequence of (25), the magnetic charge should be a pseudo-scalar. According to (9), the sign of pseudo-scalar depends on the chosen coordinate system (this can be immediate seen from the expression for the force, acting on magnetic charge in magnetic field) and, hence, the pseudo-scalar nature of magnetic charges can be considered as a proof of their existence impossibility(this issue will be, possibly, discussed later together with quantization of charges). The pseudo-scalar nature of magnetic charges, in a given case, is a direct result of using pseudo-tensor F_{B_μ} to describe an electromagnetic field generated by magnetic charges, i.e. everything, in the issue, depends on the transformation properties of electrical and magnetic fields under extended Lorentz group, consisting of Lorentz transformations, spatial rotations and inversions (changes of sign of basic vectors).

To take into account the effect on electrical charges of the electromagnetic field generated by magnetic charges, some *effective* vector-potential A_μ (corresponding to the electromagnetic field, generated by magnetic charges) is added to the four-dimensional vector potential A_e in $\mathscr{A}$ and a similar term, B_e is added to the pseudovector-potential, B_μ. Effective potentials A_μ and B_e are described by equations

$$\partial^i A^k_\mu - \partial^k A^i_\mu \equiv F^{ik}_{A_\mu} = F^{\star\, ik}_{B_\mu} = \frac{1}{2}\, e^{iklm} F_{B_\mu\, lm}, \tag{27}$$

$$\partial^i B_e^k - \partial^k B_e^i \equiv F_{B_e}^{ik} = F_{A_e}^{\star\, ik} = \frac{1}{2}\, e^{iklm} F_{A_e\, lm}. \tag{28}$$

Using the effective potential, we replace in equations the external electromagnetic field generated by magnetic (electrical) charges, which affects an electrical (magnetic) charge, by the same electromagnetic field generated by *fictitious* electrical (magnetic) charges; the necessary requirement for such replacement is that magnetic (electrical) charges $j_\mu^i(x) \equiv 0$ [$j_e^i(x) \equiv 0$, respectively] must be absent in the space-time region of interest. Because we do not consider particles bearing both electrical and magnetic charges, this requirement is always met locally, in the close vicinity of electrical (magnetic) charge. We use the variational principle to derive Lagrange-Euler differential equations (i.e. we derive them locally), hence, we may assume that the requirement above is met.

The introduction of effective potentials in some space-time region is possible for the configurations of electrical $\boldsymbol{E}$ and magnetic $\boldsymbol{H}$ fields, which may be generated both by electrical and magnetic charges, not present in the region of interest. Such configurations of electromagnetic field may be called *free* (independent of the type of charges) configurations. Electromagnetic waves, by which means the interaction of charged particles is performed, are examples of free configurations. If the necessary requirement is met, the force lines of an external electromagnetic field within a quite small space-time region are close to rectilinear, while the electromagnet field's configurations close to rectilinear may be, apparently, generated by currents of both electrical and magnetic charges, not present in the region of interest, i.e. the configurations of any electromagnetic field in the space-time region with no electrical and magnetic charges are locally free. Therefore, the effective potentials are (locally) defined in a correct way.

According to the definition, effective vector-potential A_μ^i depends on partial derivatives $\partial^l B_\mu^m$; similarly, effective pseodovector-potential B_e^i depends on partial derivatives $\partial^l A_e^m$. Consideration of the dependence of A_μ^i on partial derivatives $\partial^l B_\mu^m$ with a varying B_μ would lead to the occurrence of a term linearly depending on the value of current density j_e in the *right-hand part* of equation (15b) (for example, a partial case of a constant electromagnetic field may be considered, in this case the effective vector-potential A_μ can be easily expressed via tensor F_{B_μ}:

$$A_\mu^i = -\frac{1}{2} F_{A_\mu}^{ik} x_k = -\frac{1}{4}\, e^{iklm} F_{B_\mu\, lm} x_k; \tag{29}$$

however, in general case, the dependence of A_μ^i on partial derivatives $\partial^l B_\mu^m$ is not trivial)

and a similar term would occur in the *right-hand part* of equation (4b). This contradicts our initial assumption that the electromagnetic field generated by electrical charges and the electromagnetic field generated by magnetic charges are described independently and, hence, with varying potentials A_e and B_μ we should not vary the terms of full action, containing the effective potentials. The source-containing terms of equations (4b), (15b) make no sense to quantization of a free electromagnetic field in relativistic quantum field theory.

III. MAGNETIC MONOPOLE IN RELATIVISTIC QUANTUM FIELD THEORY

Since a distinction between action (23) and an ordinary action without magnetic charges is not important, from viewpoint of relativistic quantum field theory, the Feynman rules for the calculation of the interaction cross-sections for elementary particles interacting with electrical or magnetic charges can be formulated by generalizing the known results of relativistic quantum field theory for an electromagnetic field without magnetic charges (see, for example, [8], §24).

Operator S (S-matrix), which relates amplitudes of the initial $\Phi(-\infty)$ and the final $\Phi(\infty)$ states:

$$\Phi(\infty) = S\Phi(-\infty), \tag{30}$$

– can be expressed via chronological exponent

$$S = T\left\{\exp\left[\frac{i}{\hbar}\int \mathscr{L}_I(x) d^4x\right]\right\}, \tag{31}$$

where $\mathscr{L}_I(x)$ is Lagrangian of interaction

$$\mathscr{L}_I(x) = -\frac{1}{c^2}\left(A_e^k + A_\mu^k\right) j_{e\,k} - \frac{1}{c^2}\left(B_\mu^k + B_e^k\right) j_{\mu\,k} \tag{32}$$

and

$$j_e^k = \sum_a c e_a \bar{\psi}_a \gamma^k \psi_a, \qquad j_\mu^k = \sum_b c \mu_b \bar{\psi}_b \gamma^k \psi_b. \tag{33}$$

For the charged particle propagator we have an ordinary expression in terms of function $D^c(x-y)$ for scalar particle of mass m_n – compare with [8], (15.17), (22.9):

$$\left\langle T\{\psi_n(x)\,\bar{\psi}_n(y)\}\right\rangle = -i\left(i\gamma^k\partial_k + \frac{m_n c}{\hbar}\right) D^c(x-y). \tag{34}$$

Motion equations for free electromagnetic field [compare with (6), (17)]

$$\Box A_e^k + \partial^k \partial_l A_e^l = 0 \tag{35}$$

and

$$\Box B_\mu^k + \partial^k \partial_l B_\mu^l = 0 \tag{36}$$

have the same form, that in the usual electrodynamics. Thus, having stipulated on the potentials A_e and B_μ the Lorentz condition

$$\partial_k A_e^k \equiv 0, \qquad \partial_k B_\mu^k \equiv 0, \tag{37}$$

usual expression for chronological coupling of operators of electromagnetic field, generated by electrical charges, may be derived

$$\left\langle T\{A_e^k(x)\, A_e^l(y)\}\right\rangle = i g^{kl} D_0^c(x-y) \tag{38}$$

and analogical expression for chronological coupling of operators of electromagnetic field, generated by magnetic charges,

$$\left\langle T\{B_\mu^k(x)\, B_\mu^l(y)\}\right\rangle = i g^{kl} D_0^c(x-y)\,. \tag{39}$$

Because only vector potential A_e enters in the equation (35), and only pseudovector-potential B_μ enters in the equation(36), the chronological coupling of operators of these potentials, expressed via Green function of motion equations, is equal zero

$$\left\langle T\{A_e^k(x)\, B_\mu^l(y)\}\right\rangle = 0. \tag{40}$$

Hence, as accordingly to (27), (28) the effective potential A_μ is a linear function of B_μ, and the effective potential B_e is a linear function of A_e, chronological couplings

$$\left\langle T\{A_e^k(x)\, A_\mu^l(y)\}\right\rangle = 0, \tag{41}$$

$$\left\langle T\{A_\mu^k(x)\, B_e^l(y)\}\right\rangle = 0, \tag{42}$$

$$\left\langle T\{B_e^k(x)\, B_\mu^l(y)\}\right\rangle = 0 \tag{43}$$

are also equal zero.

In the partial case of a constant electromagnetic field, generated by magnetic charges, when for A_μ expression (29):

$$A_\mu^k = -\frac{1}{2} F_{A_\mu}^{kl} x_l = -\frac{1}{4}\, e^{klmn} F_{B_\mu\, mn} x_l = -\frac{1}{4} e^{klmn} \left(\frac{\partial B_{\mu\, n}}{\partial x^m} - \frac{\partial B_{\mu\, m}}{\partial x^n} \right) x_l \tag{44}$$

– may be used, the chronological coupling of operators A_μ and B_μ is equal

$$\begin{aligned} \left\langle T\{A_\mu^k\,(x)\,B_\mu^q\,(y)\}\right\rangle &= -\frac{1}{4} e^{klmn} x_l \delta_n^q \frac{\partial D_0^c\,(x-y)}{\partial x^m} \\ &\quad + \frac{1}{4} e^{klmn} x_l \delta_m^q \frac{\partial D_0^c\,(x-y)}{\partial x^n} \\ &= \frac{1}{2} e^{klqm} x_l \frac{\partial D_0^c\,(x-y)}{\partial x^m}. \end{aligned} \tag{45}$$

Similarly, for a constant electromagnetic field, generated by electrical charges,

$$\left\langle T\{A_e^k\,(x)\,B_e^q\,(y)\}\right\rangle = -\frac{1}{2} e^{klqm} y_l \frac{\partial D_0^c\,(x-y)}{\partial y^m}. \tag{46}$$

[1] P. A. M. Dirac, Proc. Roy. Soc., A133, 60 (1931).

[2] P. A. M. Dirac, Scientific American, 208, No. 5, 45 (1963).

[3] K. A. Milton, Reports on Progress in Physics, 69 (6), 16371711 (2006).

[4] F. Rohrlich, Phys. Rev. 150, 1104, (1966)

[5] D. Zwanziger, Phys. Rev. 3, 880 (1971).

[6] S. Weinberg, Phys. Rev. 138, B988 (1965).

[7] L. D. Landau, E. M. Lifshitz, *The Classical Theory of Fields*, Fourth Edition: Volume 2 (Course of Theoretical Physics Series), (Pergamon Press, Oxford, 2000).

[8] N. N. Bogoliubov, D. V. Shirkov, *Introduction to the theory of quantized fields*, (Wiley, New York, 1980).

[9] J. . Schouten, *Tensor analysis for physicists*, (Clarendon Press, Oxford, 1951).

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Abstract

The paper offers the full action for an electromagnetic field with electrical and magnetic charges; Feynman laws are formulated for the calculation of the interaction cross-sections for electrically and magnetically charged particles on the base of offered action within relativistic quantum field theory. Derived with formulated Feynman rules cross-section of the interaction between an elementary particle with magnetic charge and an elementary particle with electrical charge proves to be equal zero.

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I. INTRODUCTION

The papers [1], [2] by Dirac, in which the existence of magnetic charges was related to quantization of electrical charges, were followed by intensively searching for magnetic monopoles, however, the search has not yet led to success (see, for example, [3]). All the attempts of such search were complicated by the lack of a satisfactory theoretical description of interactions between ordinary electrical and magnetic charges. Moreover, Rohrlich in [4] stated the "falsehood of variational principle for the full theory of . . . electrical and magnetic point charges". Later, Zwanziger in [5] proposed the full action, including an arbitrary constant 4-vector n, with the electromagnetic field propagator being dependent of arbitrary vector n; as a result, the cross-section of the interaction of an elementary particle with magnetic charge and an elementary particle with electrical charge proves to be dependent of the arbitrary vector n (terms in the interaction cross-section, containing arbitrary vector n, can vanish only for interactions of particles with same type of charge).

For the description of the electromagnetic field with electrical and magnetic charges below we use *two* four-dimensional vector potentials A_e and B_g. Apparently, for the first time two four-dimensional vector potentials were used for the description of the electromagnetic field with electrical and magnetic charges in the Cabibbo and Ferrari article [6] – see, for example, Singleton review [7]. While there is not enough information, we are inclined to suppose, that quantums of the A_e-field and the B_g-field are the same. Another possibility is discussed in works [8], [9].

In this paper we propose the full action for an electromagnetic field with electrical and magnetic charges. Particles with both electrical and magnetic charges are not considered (compare with the statement by Weinberg in [10], that "a magnetic monopole cannot bear a normal charge"). Offered full action does not contain arbitrary constant 4-vector n. We formulate Feynman laws for the calculation of the interaction cross-sections for electrically and magnetically charged particles on the base of offered action within relativistic quantum field theory. It is shown, that derived with formulated Feynman rules cross-section of the interaction between an elementary particle with magnetic charge and an elementary particle with electrical charge is equal zero. Possibly, this explains the failure of the search of magnetic monopoles with ordinary elementary particle detectors.

The expressions below are written in Gaussian units; used denotations are close to that

in [11] and [12]; in particular, coordinate indices of four-dimensional vectors and tensors are denoted by latin letters and have values from 0 to 3; a metrics is considered to be defined with diagonal metric tensor: $g^{ik} = 0$ at $i \neq k$, $g^{00} = 1$, $g^{11} = g^{22} = g^{33} = -1$; the same upper and lower coordinate indices of four-dimensional tensors always imply summation; if some indices are not coordinate tensor indices, the summation symbol is explicitly shown, if necessary; the four-dimensional coordinate is $x^i = (ct, \boldsymbol{r})$, $x_i = (ct, -\boldsymbol{r})$, $x^i x_i = c^2t^2 - \boldsymbol{r}^2$, where c is the light speed constant, t is time, $\boldsymbol{r}$ is three-dimensional radius-vector.

II. MAGNETIC MONOPOLE IN CLASSIC FIELD THEORY

It would appear reasonable that the properties of magnetic charges and the electromagnetic field generated by them would be similar to the properties of electrical charges and the electromagnetic field generated by them: in particular, the force lines of the magnetic field, generated by magnetic charges, start from/end in the magnetic charges, while the force lines of the electrical field generated by currents of magnetic charges are closed, i.e. the existence of magnetic charges, in a sense, restores *symmetry* between the magnetic and electrical fields. Apparently, the equations describing the electromagnetic field generated by magnetic charges (currents) should be similar to the equations describing the electromagnetic field generated by electrical charges (currents). At the same time, in view of the topological difference between the electromagnetic field generated by magnetic charges and the one generated by electrical charges they should be described *separately* in equations.

The electromagnetic field, generated by the current density of electrical charges $\{e_a\}$ – j_e,– can be described using the antisymmetric four-dimensional tensor of the second rank F_{A_e}, which can be represented via the four-dimensional vector potential, $A_e^i \overset{\text{def}}{=} (\varphi_e, \boldsymbol{A}_e)$ in the following form:

$$F_{A_e}^{ik} \overset{\text{def}}{=} \frac{\partial A_e^k}{\partial x_i} - \frac{\partial A_e^i}{\partial x_k} = \partial^i A_e^k - \partial^k A_e^i. \tag{1}$$

The relation between components of tensor F_{A_e} and components of (three-dimensional) vectors of electrical field $\boldsymbol{E}$ and magnetic field $\boldsymbol{H}$ is described as

$$E_e^{\alpha} = -F_{A_e}^{0\alpha} \quad (\alpha = 1, 2, 3)\,; \quad H_e^1 = -F_{A_e}^{23}, \quad H_e^2 = -F_{A_e}^{31}, \quad H_e^3 = -F_{A_e}^{12} \tag{2}$$

or

$$F^{ik}_{A_e} = \begin{pmatrix} 0 & -E^1_e & -E^2_e & -E^3_e \\ E^1_e & 0 & -H^3_e & H^2_e \\ E^2_e & H^3_e & 0 & -H^1_e \\ E^3_e & -H^2_e & H^1_e & 0 \end{pmatrix} \qquad F_{A_e\, ik} = \begin{pmatrix} 0 & E^1_e & E^2_e & E^3_e \\ -E^1_e & 0 & -H^3_e & H^2_e \\ -E^2_e & H^3_e & 0 & -H^1_e \\ -E^3_e & -H^2_e & H^1_e & 0 \end{pmatrix}. \tag{3}$$

The ordinary Maxwell's equation system, describing an electromagnetic field (in vacuum), generated by the current density of electrical charges j_e, can be written as

$$\frac{\partial F^{ik}_{A_e}}{\partial x_l} + \frac{\partial F^{kl}_{A_e}}{\partial x_i} + \frac{\partial F^{li}_{A_e}}{\partial x_k} = 0, \tag{4a}$$

$$\frac{\partial F^{ik}_{A_e}}{\partial x^k} = -\frac{4\pi}{c} j^i_e. \tag{4b}$$

Since the result of action of operator $\frac{\partial^2}{\partial x^i \partial x^k}$ (symmetric with respect to indices i, k) on antisymmetric tensor $F^{ik}_{A_e}$ (similar to any convolution of symmetric tensor with the antisymmetric one) identically equals zero, the continuity equation describing the charge conservation law follows from (4b):

$$\frac{\partial j^i_e}{\partial x^i} = 0. \tag{5}$$

In quantum field theory, equation (4b) is replaced, according to definition (1), by the following equation:

$$-\partial_k \partial^k A^i_e + \partial_k \partial^i A^k_e \overset{\text{def}}{=} \Box A^i_e + \partial_k \partial^i A^k_e = -\frac{4\pi}{c} j^i_e. \tag{6}$$

In the equation (4a) instead of tensor F_{A_e} *pseudo-tensor*

$$F^{\star\, ik}_{A_e} \overset{\text{def}}{=} \frac{1}{2!}\, e^{iklm} F_{A_e\, lm}, \tag{7}$$

dual to tensor F_{A_e}, may be used, where e^{iklm} is the absolutely antisymmetric, unit, four-dimensional pseudo-tensor of the fourth rank with weight $W = +1$, which components change their signs with permutation of any two indices and

$$e^{0123} \overset{\text{def}}{=} +1, \tag{8}$$

by definition (pseudo-tensor e_{iklm} has weight $W = -1$ and $e_{0123} = -1$). With x coordinate system replaced by $\bar{x}$ coordinate system (caused by changes in the basis of n-dimensional

vector space), components of pseudo-tensor T of weight W, which is r-times contra-variant and s-times co-variant, are transformed according to the law (see, for example, [13]):

$$\bar{T}^{k_1 k_2 \dots k_r}_{k'_1 k'_2 \dots k'_s} \stackrel{\text{def}}{=} \frac{\partial \bar{x}^{k_1}}{\partial x^{i_1}} \frac{\partial \bar{x}^{k_2}}{\partial x^{i_2}} \cdots \frac{\partial \bar{x}^{k_r}}{\partial x^{i_r}} \frac{\partial x^{i'_1}}{\partial \bar{x}^{k'_1}} \frac{\partial x^{i'_2}}{\partial \bar{x}^{k'_2}} \cdots \frac{\partial x^{i'_s}}{\partial \bar{x}^{k'_s}} T^{i_1 i_2 \dots i_r}_{i'_1 i'_2 \dots i'_s} J^W. \tag{9}$$

In contrast to the ordinary tensor's component transformation, there is a factor in the form of Jacobian of coordinate transformation

$$J = \frac{\partial \left(x^0, x^1, \dots, x^{n-1}\right)}{\partial \left(\bar{x}^0, \bar{x}^1, \dots, \bar{x}^{n-1}\right)} \qquad |J| = \sqrt{-\det\left(g^{ik}\right)} = 1 \tag{10}$$

of power W in equation (9). The weight of the product of two tensors equals the summarized weight of tensors of each factor and convolution of each pair of indices, consisting of the identical upper and lower indices, does not change the weight of tensor; all pseudo-tensors considered in the paper have weight ± 1; in particular, a magnetic charge is a pseudo-scalar, see below, and, hence, we are not far from the fore-quoted Weinberg statement.

The left-hand part of equation (4a) represents by itself a tensor of the third rank, which is antisymmetric in all indices. Having lowered indices of this tensor, multiplied by e^{mlik} and performed convolution with respect to three pairs of identical indices, we obtain equation

$$\frac{\partial F^{\star\, ml}_{A_e}}{\partial x^l} = 0. \tag{11}$$

The relation between components of pseudo-tensor $F^{\star}_{A_e}$, which is dual to F_{A_e}, and components of (three-dimensional) vectors of electrical field $\boldsymbol{E}$ and magnetic field $\boldsymbol{H}$ is described as

$$H^{\alpha}_e = -F^{\star\, 0\alpha}_{A_e} \quad (\alpha = 1, 2, 3)\,; \quad E^1_e = -F^{\star\, 23}_{A_e}, \quad E^2_e = -F^{\star\, 31}_{A_e}, \quad E^3_e = -F^{\star\, 12}_{A_e} \tag{12}$$

or

$$F^{\star\, ik}_{A_e} = \begin{pmatrix} 0 & -H^1_e & -H^2_e & -H^3_e \\ H^1_e & 0 & -E^3_e & E^2_e \\ H^2_e & E^3_e & 0 & -E^1_e \\ H^3_e & -E^2_e & E^1_e & 0 \end{pmatrix} \qquad F^{\star}_{A_e\, ik} = \begin{pmatrix} 0 & H^1_e & H^2_e & H^3_e \\ -H^1_e & 0 & -E^3_e & E^2_e \\ -H^2_e & E^3_e & 0 & -E^1_e \\ -H^3_e & -E^2_e & E^1_e & 0 \end{pmatrix}, \tag{13}$$

i.e. in pseudo-tensor $F^{\star}_{A_e}$, as compared to tensor F_{A_e}, components of (three-dimensional) vectors of the electrical and magnetic fields change places. So, it would appear reasonable, that the electromagnetic field generated by the current density of magnetic charges

$\{g_b\}$ – j_g could be described using the antisymmetric four-dimensional pseudo-tensor of the second rank, F_{B_g}, which is structurally similar to pseudo-tensor $F^{\star}_{A_e}$ and described via four-dimensional pseudovector-potential $B_g^i \stackrel{\text{def}}{=} (\psi_g, \boldsymbol{B}_g)$ in the following way:

$$F_{B_g}^{ik} \stackrel{\text{def}}{=} \frac{\partial B_g^k}{\partial x_i} - \frac{\partial B_g^i}{\partial x_k} = \partial^i B_g^k - \partial^k B_g^i. \tag{14}$$

Then, instead of equations (4)-(6), (11), to describe the electromagnetic field (in vacuum), generated by the current density of magnetic charges j_g, one could use the following equations:

$$\frac{\partial F_{B_g}^{ik}}{\partial x_l} + \frac{\partial F_{B_g}^{kl}}{\partial x_i} + \frac{\partial F_{B_g}^{li}}{\partial x_k} = 0, \tag{15a}$$

$$\frac{\partial F_{B_g}^{ik}}{\partial x^k} = -\frac{4\pi}{c} j_g^i. \tag{15b}$$

$$\frac{\partial j_g^i}{\partial x^i} = 0, \tag{16}$$

$$-\partial_k \partial^k B_g^i + \partial_k \partial^i B_g^k = \Box B_g^i + \partial_k \partial^i B_g^k = -\frac{4\pi}{c} j_g^i, \tag{17}$$

$$\frac{\partial F_{B_g}^{\star\, ml}}{\partial x_l} = 0. \tag{18}$$

To obtain a closed equation system, describing the electromagnetic field together with electrical and magnetic charges, equations (4b) and (15b) must be complemented with motion equations for electrical charges

$$m_a\, c\, \frac{du_a^i}{ds} = \frac{e_a}{c} \left(F_{A_e}^{ik} + F_{B_g}^{\star\, ik} \right) u_{a\, k} \tag{19}$$

and magnetic charges

$$m_b\, c\, \frac{du_b^i}{ds} = \frac{g_b}{c} \left(F_{A_e}^{\star\, ik} + F_{B_g}^{ik} \right) u_{b\, k}. \tag{20}$$

In equation (19), m_a is the a-th electrical charge's mass, $ds = \sqrt{dx^i dx_i}$,

$$u_a^i \stackrel{\text{def}}{=} \frac{dx_a^i}{ds} \tag{21}$$

is the dimensionless four-dimensional velocity of the a-th electrical charge. The same notations are used in equation (20). In the right-hand parts of equations (19), (20) in parentheses

the electrical and magnetic fields, generated by electrical and magnetic charges, are summarized, according to the superposition law.

Equations (19), (20), (4b), (15b) can be obtained, basing on the requirement of

$$\delta\mathscr{A} = 0 \tag{22}$$

for variation of the full action

$$\begin{aligned}\mathscr{A} = &-\sum_a \int m_a c ds - \frac{1}{c^2}\int \left(A_e^i + A_g^i\right) j_{e\,i}\, d^4x - \frac{1}{16\pi c}\int F_{A_e\,ik}\, F_{A_e}^{ik} d^4x \\ &-\sum_b \int m_b c ds - \frac{1}{c^2}\int \left(B_g^i + B_e^i\right) j_{g\,i}\, d^4x - \frac{1}{16\pi c}\int F_{B_g\,ik}\, F_{B_g}^{ik} d^4x,\end{aligned} \tag{23}$$

if we vary trajectories of electrical charges, trajectories of magnetic charges, vector-potential A_e and pseudovector-potential B_g. Integration in first and fourth terms in the right-hand part of equation (23) is performed with respect to the world lines between two given events, in the rest terms integration is performed with respect to four-dimensional volume; variations of functions are assumed to be equal zero on the integration domain boundary. Instead of the sum of third and sixth terms in the right-hand part of equation (23) expression

$$-\frac{1}{16\pi c}\int \left(F_{A_e\,ik} + F^{\star}_{B_g\,ik}\right)\left(F_{A_e}^{ik} + F_{B_g}^{\star\,ik}\right) d^4x, \tag{24}$$

may be used, as variations of products $F_{A_e\,ik}F_{B_g}^{\star\,ik}$ and $F^{\star}_{B_g\,ik}F_{A_e}^{ik}$ are equal zero because of (11) and (18). To vary trajectories of electrical charge e_a, we use the expression, describing the current density of this charge via the delta-function of three-dimensional argument:

$$j_{e_a}^i = e_a \delta\left(\boldsymbol{r} - \boldsymbol{r}_a\right) \frac{dx^i}{dt}, \tag{25}$$

a similar expression for the current density of a magnetic charge

$$j_{g_b}^i = e_b \delta\left(\boldsymbol{r} - \boldsymbol{r}_b\right) \frac{dx^i}{dt} \tag{26}$$

is used to vary the magnetic charge trajectories;

$$j_e^i = \sum_a j_{e_a}^i, \quad j_g^i = \sum_b j_{g_b}^i. \tag{27}$$

Because of (15b) density of current j_g is a pseudo-vector and, in consequence of (26), the magnetic charge should be a pseudo-scalar. According to (9), the sign of pseudo-scalar depends on the chosen coordinate system (this can be immediate seen from the expression for the force, acting on magnetic charge in magnetic field) and, hence, the pseudo-scalar

nature of magnetic charges can be considered as a proof of their existence impossibility (this issue will be, possibly, discussed later together with quantization of charges). The pseudo-scalar nature of magnetic charges, in a given case, is a direct result of transformation properties of electrical and magnetic fields under extended Lorentz group, consisting of Lorentz transformations, spatial rotations and inversions (changes of sign of basic vectors).

To take into account the effect on electrical charges of the electromagnetic field, generated by magnetic charges, some *effective* vector-potential A_g (corresponding to the electromagnetic field, generated by magnetic charges) is added to the four-dimensional vector potential A_e in $\mathscr{A}$ and a similar term B_e is added to the pseudovector-potential B_g. Effective potentials A_g and B_e are described by equations

$$\partial^i A_g^k - \partial^k A_g^i \stackrel{\text{def}}{=} F_{A_g}^{ik} = F_{B_g}^{\star\, ik} = \frac{1}{2}\, e^{iklm} F_{B_g\, lm}, \tag{28}$$

$$\partial^i B_e^k - \partial^k B_e^i \stackrel{\text{def}}{=} F_{B_e}^{ik} = F_{A_e}^{\star\, ik} = \frac{1}{2}\, e^{iklm} F_{A_e\, lm}. \tag{29}$$

Using the effective potential, we replace in equations the external electromagnetic field, generated by magnetic (electrical) charges, which affects an electrical (magnetic) charge, by the same electromagnetic field, generated by *fictitious* electrical (magnetic) charges; the requirement for such replacement is that magnetic (electrical) charges must be absent in the space-time region of interest [electromagnetic field must not have singularities due to magnetic (electrical) charges]:

$$j_g^i\,(x) \equiv 0 \tag{30}$$

or

$$j_e^i\,(x) \equiv 0, \tag{31}$$

respectively. Because we do not consider particles bearing both electrical and magnetic charges, this requirement is always met locally, in the close vicinity of electrical (magnetic) charge. As we use the variational principle to derive Lagrange-Euler *differential* equations, i.e. locally, we may assume, that these requirements are met. If requirement (30) is met, external derivation of antisymmetric four-dimensional tensor of the second rank $F_{A_g}^{ik}$

$$\frac{\partial F_{A_g}^{ik}}{\partial x_l} + \frac{\partial F_{A_g}^{kl}}{\partial x_i} + \frac{\partial F_{A_g}^{li}}{\partial x_k}, \tag{32}$$

written in the form [compare with (4a), (11)]

$$\frac{\partial F^{\star\, ml}_{A_g}}{\partial x^l} = \frac{\partial F^{ml}_{B_g}}{\partial x^l} = 0, \tag{33}$$

equals (identically) zero by virtue of (15b) and (28), and it is possible to write explicit expression for effective potential A_g, using Poincaré Lemma (see, for example, [14] or [15]):

$$A^k_g(x) = \int\limits_0^1 \zeta F^{ik}_{A_g}(\zeta x)\, x_i\, d\zeta = \frac{1}{2}\int\limits_0^1 \zeta e^{iklm} F_{B_g\, lm}(\zeta x)\, x_i\, d\zeta. \tag{34}$$

Analogous expression for effective potential B_e

$$B^k_e(x) = \int\limits_0^1 \zeta F^{ik}_{B_e}(\zeta x)\, x_i\, d\zeta = \frac{1}{2}\int\limits_0^1 \zeta e^{iklm} F_{A_e\, lm}(\zeta x)\, x_i\, d\zeta \tag{35}$$

may be written, if requirement (31) is met. We might do not introduce effective potentials A_g and B_e, but simply use expressions (34) and (35) in the expression for the full action (23); i.e. the action (23) is ordinary normal action – compare with the Rohrlich statement upper.

According to the definition, effective vector-potential A^i_g depends on partial derivatives $\partial^l B^m_g$; similarly, effective pseodovector-potential B^i_e depends on partial derivatives $\partial^l A^m_e$. Consideration of the dependence of A^i_g on partial derivatives $\partial^l B^m_g$ with a varying B_g would lead to the occurrence of a term, linearly depending on the value of current density j_e, in the *right-hand part* of equation (15b), and a similar term, linearly depending on the value of current density j_g, would occur in the *right-hand part* of equation (4b). This contradicts our initial assumption that the electromagnetic field generated by electrical charges and the electromagnetic field generated by magnetic charges are described independently and, hence, with varying potentials A_e and B_g we should not vary the terms of full action, containing the effective potentials. In any case, the source-containing terms of equations (4b), (15b) make no sense to quantization of a free electromagnetic field in relativistic quantum field theory.

III. MAGNETIC MONOPOLE IN RELATIVISTIC QUANTUM FIELD THEORY

Since a distinction between action (23) and an ordinary action without magnetic charges is not important from viewpoint of relativistic quantum field theory, the Feynman rules for

the calculation of the interaction cross-sections for elementary particles with electrical or magnetic charges can be formulated by generalization of known results of relativistic quantum field theory for an electromagnetic field without magnetic charges – see, for example, [12], § 24 or [16], § 77.

Operator S (S-matrix), which relates amplitudes of the initial $\Phi(-\infty)$ and the final $\Phi(\infty)$ states:

$$\Phi(\infty) = S\Phi(-\infty), \tag{36}$$

– can be expressed via chronological exponent

$$S = T\left\{\exp\left[\frac{i}{\hbar}\int \mathscr{L}_I(x) d^4x\right]\right\}, \tag{37}$$

where $\mathscr{L}_I(x)$ is lagrangian of interaction

$$\mathscr{L}_I(x) = -\frac{1}{c^2}\left(A_e^k + A_g^k\right) j_{e\,k} - \frac{1}{c^2}\left(B_g^k + B_e^k\right) j_{g\,k} \tag{38}$$

and

$$j_e^k = \sum_a c e_a \bar{\psi}_a \gamma^k \psi_a, \qquad j_g^k = \sum_b c g_b \bar{\psi}_b \gamma^k \psi_b. \tag{39}$$

For propagator of fermions with magnetic charge we have the usual expression in terms of causal function $D^c(x-y)$ for scalar particle of mass m_b – compare with [12], (15.17), (22.9):

$$\left\langle T\left\{\psi_b(x)\,\bar{\psi}_b(y)\right\}\right\rangle = -i\left(i\gamma^k\partial_k + \frac{m_b c}{\hbar}\right) D^c(x-y). \tag{40}$$

Motion equations for free electromagnetic field [compare with (6), (17)]

$$\Box A_e^k + \partial^k \partial_l A_e^l = 0 \tag{41}$$

and

$$\Box B_g^k + \partial^k \partial_l B_g^l = 0 \tag{42}$$

have the same form, that in the usual electrodynamics. Thus, the usual expression for chronological coupling of operators of electromagnetic field, generated by electrical charges, may be derived (in the Feynman gauge)

$$\left\langle T\left\{A_e^k(x)\,A_e^l(y)\right\}\right\rangle = i4\pi g^{kl} D_0^c(x-y), \tag{43}$$

and analogical expression may be derived for chronological coupling of operators of electromagnetic field, generated by magnetic charges,

$$\left\langle T\left\{B_g^k(x)\, B_g^l(y)\right\}\right\rangle = i4\pi g^{kl} D_0^c(x-y)\,. \tag{44}$$

Because only vector potential A_e enters in the equation (41), and only pseudovector-potential B_g enters in the equation (42), the chronological coupling of operators of these potentials, expressed via Green function of motion equations, is equal zero

$$\left\langle T\left\{A_e^k(x)\, B_g^l(y)\right\}\right\rangle = 0. \tag{45}$$

Hence, as accordingly to (28), (29) the effective potential A_g is a linear function of B_g, and the effective potential B_e is a linear function of A_e, chronological couplings

$$\left\langle T\left\{A_e^k(x)\, A_g^l(y)\right\}\right\rangle = 0, \tag{46}$$

$$\left\langle T\left\{A_g^k(x)\, B_e^l(y)\right\}\right\rangle = 0, \tag{47}$$

$$\left\langle T\left\{B_e^k(x)\, B_g^l(y)\right\}\right\rangle = 0 \tag{48}$$

are also equal zero.

Using expression (34) for $A_g(x)$, we obtain for the chronological coupling of operators $A_g(0)$ and $B_g(y)$:

$$\left\langle T\left\{A_g^k(x)\, B_g^q(y)\right\}\right\rangle\Big|_{x=0} = \left\langle T\left\{\left[e^{klmn} x_l \int\limits_0^1 \zeta \frac{\partial B_{g\,m}(\zeta x)}{\partial x^n}\, d\zeta\right] B_g^q(y)\right\}\right\rangle\Bigg|_{x=0} = 0. \tag{49}$$

By virtue of the homogeneity of the space-time and the arbitrariness of y in (49) we can conclude, that

$$\left\langle T\left\{A_g^k(x)\, B_g^q(y)\right\}\right\rangle \equiv 0. \tag{50}$$

Similarly, for electromagnetic field, generated by electrical charges,

$$\left\langle T\left\{A_e^k(x)\, B_e^q(y)\right\}\right\rangle \equiv 0. \tag{51}$$

Element of the S-matrix, corresponding to the interaction of elementary particle with magnetic charge and elementary particle with electrical charge, is a sum of terms, proportional to the chronological coupling (50) or (51), and so it proves to be equal zero; therefore,

interaction cross-section of an elementary particle with magnetic charge and an elementary particle with electrical charge is equal zero. For the magnetic monopoles search one may try to use detectors of weak magnetic field instead of ordinary detectors of elementary particles, based on the interaction of moving (electrically) charged particles with detector's electrical charges.

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[1] P. A. M. Dirac, Proc. R. Soc. Lond., **A133**(821), 60-72 (1931)

[2] P. A. M. Dirac, Sci. Am., **208**(5), 45-53 (1963)

[3] K. A. Milton, Rep. Prog. Phys., **69**(6), 1637-1711 (2006)

[4] F. Rohrlich, Phys. Rev., **150**, 1104-1111 (1966)

[5] D. Zwanziger, Phys. Rev., **D3**, 880-891 (1971)

[6] N. Cabibbo and E. Ferrari, Nuovo Cimento, **23**, 1147-1154 (1962)

[7] D. Singleton, Am. J. Phys., **64**, 452-458 (1996)

[8] D. Singleton, Int. J. Theor. Phys., **34**, 37-46 (1995); D. Singleton, Int. J. Theor. Phys., **35**, 2419-2426 (1996)

[9] P. Castelo Ferreira, J. Math. Phys., **47**, 072902 (2006); P. Castelo Ferreira, Europhys. Lett., **79**, 20004 (2007)

[10] S. Weinberg, Phys. Rev., **B138**, 988-1002 (1965)

[11] L. D. Landau, E. M. Lifshitz: The Classical Theory of Fields, Fourth Edition: Volume 2 (Course of Theoretical Physics Series). Pergamon Press, Oxford (2000)

[12] N. N. Bogoliubov, D. V. Shirkov: Introduction to the Theory of Quantized Fields. Wiley, New York (1980)

[13] J. A. Schouten: Tensor analysis for physicists. Clarendon Press, Oxford (1951)

[14] S. Lang: Introduction to Differentiable Manifolds. Springer, New York (2002)

[15] B. P. Kosyakov: Introduction to the Classical Theory of Particles and Fields. Springer, Berlin

(2007)

[16] V. B. Berestetskii, E. M. Lifshitz, L. P. Pitaevskii: Quantum Electrodynamics, 2nd Edition (L. D. Landau, E. M. Lifshitz, Course of Theoretical Physics, Volume 4). Butterworth-Heinemann, Oxford (1982)